Docket No. GB920030048US1 *PATENT*

IN THE UNITED STATES PATENT AND TRADEMARK OFFICE

In re application of: **Chessell et al.**	§	
	§	Group Art Unit: **2163**
Serial No. **10/798,920**	§	
	§	Examiner: **Lie, Angela M.**
Filed: **March 11, 2004**	§	
	§	
For: **Profiling Data in a Data Store**	§	
	§	

Commissioner for Patents
P.O. Box 1450
Alexandria, VA 22313-1450

35525
PATENT TRADEMARK OFFICE
CUSTOMER NUMBER

REPLY BRIEF (37 C.F.R. 41.41)

This Reply Brief is submitted in response to the Examiner's Answer mailed on November 16, 2007.

No fees are believed to be required to file a Reply Brief. If any fees are required, I authorize the Commissioner to charge these fees which may be required to IBM Corporation Deposit Account No. 09-0447.

RESPONSE TO EXAMINER'S ANSWER

I. Refutation of the Examiner's First Response

In response to Appellants pointing out that the objection to claim 1 is in error, the Examiner responds that:

> The Appellant's first argument has not been found persuasive. The Examiner would like to note that the source of the objection is not derived from the claimed limitation, "empty set", but the phrase that there is an empty intersection. It is well known in the art, that when a person attempts to obtain a result by intersecting two sets (for instance, searching for same elements contained in both sets or matching profile with the respective resource), the intersection of those two sets is a clear indication that there is at least one element that those two sets share in common. Consequently, if those two sets do not comprise a common element, the intersection cannot be made, instead commonly spoken, the intersection does not exist. Stating that there is an empty intersection is against common mathematical standards, since the intersection is not described by two contours of the sets encircling each other, instead it is defined by the elements that those two sets share in common, or two element that correspond to each other.

Examiner's Answer of November 16, 2007, pp. 7-8.

Claim 1 is as follows:

> 1. A method for extracting data from a data store comprising a first set of one or more data items, the method comprising the steps of:
>
> creating a selected set comprising a second set of one or more data items in accordance with a selection rule;
>
> creating a profile of the data store, the profile comprising a profile rule defining a profile set, wherein the profile set comprises a third set of one or more data items in accordance with the profile rule;
>
> responsive to a determination that an intersection of the selected set and the profile set is non-empty, extracting a fourth set of one or more data items from the data store in accordance with the selection rule; and
>
> responsive to a determination that the intersection of the selected set and the profile set is empty, providing an indication that the data store does not include data items in the selected set.

The Examiner asserts that the source of the rejection derives from the claimed phrase that the claim contains a phrase directed to an empty intersection. Appellants assume that the Examiner objects to the claimed phrase, "responsive to a determination that the intersection of the selected set

and the profile set is empty." The Examiner then asserts that "it is well known" that when a person attempts to obtain a result by intersecting two sets, the intersection of those two sets "is a clear indication" that the intersection contains at least one element that the two sets share. The Examiner asserts that if the two sets do not have a common element, the intersection cannot be made and that the intersection does not exist. The Examiner asserts that "an empty intersection" is against common mathematical standards, because the intersection is defined by the elements that the two sets share.

However, the Examiner's unsupported assertion regarding "mathematical standards" is manifestly contrary to the knowledge of one of ordinary skill. An intersection of two sets can be empty, such as when neither set shares an element in common or when one or both sets are empty. The Examiner's error lies in the assumption that at least one element must be common to the two sets. This assumption is incorrect, and contrary to the understanding of those of ordinary skill.

In mathematics, an intersection can be constructed by determining which members that two sets, A and B, have in common. The intersection of A and B, denoted by $A \cap B$, is the set of all things which are members of both A and B. If $A \cap B = \varnothing$ (the empty set), then the intersection of A and B is said to be empty, or disjoint.

For further edification, the Board is invited to review the attached lecture note by Herman Bennet for the Spring, 2006 lecture in Set and Probability Theory at the *Massechusets Institute of Technology (MIT)*. Note that item 10 on page 4 of this document shows that the intersection of two sets can, indeed, be empty. The Board is also invited to review a variety of web-sites describing set theory, including web01.shu.edu/projects/reals/logic/notation.html (*Seton Hall University*) and *mathepi.com*. Thus, the evidence provided by Appellants is not an isolated example, but rather indicative of the wide understanding of those skilled in mathematics.

As shown above, the unsupported assertion that the intersection of two sets cannot be empty is manifestly incorrect. Therefore, this objection is erroneous and should be overturned.

Note that the anticipation rejection relies on the incorrect assertion that the intersection of two sets cannot be empty. Thus, the anticipation rejection is also wrong and should be overturned.

II. Refutation of the Examiner's Second Response

In response to Appellant's argument that *Siefert*, Automated Resource Management System, U.S. Patent Application Publication 2002/0194179 (December 19, 2002) (hereinafter

"*Siefert*") does not teach a level of descriptiveness, as claimed, the Examiner states that:

> The Appellant's second argument has not been found persuasive. *The Examiner maintains that Siefert* ***indirectly*** *teaches the "level of descriptiveness"*, by disclosing that the profile of the resource is represented by a descriptive title and resource location information (paragraphs 58 and 99). Furthermore, the profile also comprises description of the data contained in the corresponding resource. Note that the requirement for the specific data being present to create a profile is considered a "level of descriptiveness" since every single of those pieces of data (i.e. tile, location a detailed description) is required to form a profile. Consequently the above-defined rule/requirement is used to create profiles (paragraph 58, i.e. profile set, wherein the collection of profiles comprises at least two profiles).
>
> Moreover the Examiner would also like to note that the Appellant does not define the phrase "profile rule" instead multiple examples are presented. Therefore the Examiner is entitled to presume the broadest, reasonable interpretation.

Examiner's Answer of November 16, 2007, p. 9 (emphasis supplied).

When asserting an anticipation rejection, the Examiner is not entitled to rely on the "indirect" teachings of the reference. Anticipation focuses on whether a claim reads on the product or process a prior art reference discloses, not on what the reference broadly teaches. *Kalman v. Kimberly-Clark Corp.*, 713 F.2d 760, 218 U.S.P.Q. 781 (Fed. Cir. 1983). Indeed, "to establish inherency, the extrinsic evidence 'must make clear that the missing descriptive matter is ***necessarily*** present in the thing described in the reference, and that it would be so recognized by persons of ordinary skill. *Inherency, however, may not be established by probabilities or possibilities.* The mere fact that a certain thing may result from a given set of circumstances is not sufficient'." *In re Robertson*, 169 F.3d 743, 745, 49 USPQ2d 1949, 1950-51 (Fed. Cir. 1999) (emphasis supplied).

In this case, the Examiner has admitted that *Siefert* only "indirectly" teaches a "level of descriptiveness." Thus, under the standards of *Kalman* and *In re Robertson*, the Examiner's admission has effectively proved that the anticipation rejection is in error and should be overturned.

Additionally, while the Examiner is correctly entitled to presume the broadest reasonable interpretation of the claims, such interpretation must be *reasonable*. In the case at hand, the Examiner has no reasonable basis to assume that the teachings of *Siefert* provide for a "level of descriptiveness." *Siefert* only states that the profile contains a description of the resource. In paragraph 0059, *Siefert* states that the description is a title. In paragraph 0114, *Siefert* states that the

description can be where the resource is located. However, contrary to the Examiner's statement, *Siefert* provides no "level" of descriptiveness.

Additionally, the Examiner has no *reasonable* basis to assume that if *Siefert* did teach a "level of descriptiveness," such a feature would be equivalent to a "profile rule" or "profile set," as claimed. A description is not a rule, so the teachings of *Siefert* and the claimed terms are not equivalent. The Examiner may not twist the meaning of claimed terms beyond their reasonable and ordinary meaning, as possibly modified by the specification. In the case at hand, because no basis exists to assume that a description is a rule, the Examiner has twisted the meaning of the claimed terms beyond their reasonable and ordinary meaning, a meaning not contradicted by the specification.

Hence, *Siefert* does not teach the claimed feature of, "creating a profile of the data store, the profile comprising a profile rule defining a profile set, wherein the profile set comprises a third set of one or more data items in accordance with the profile rule," as recited in claim 1. Accordingly, *Siefert* does not anticipate claim 1.

III. Refutation of the Examiner's Third Response

In response to Appellant's argument that *Siefert* does not teach, "responsive to a determination that the intersection of the selected set and the profile set is empty, providing an indication that the data store does not include data items in the selected set," the Examiner states that:

> The Examiner does not find the third argument persuasive. As addressed above (response to the first argument), the "empty intersection" i.e. non-existing intersection is an indication that there are no results available, in other words if there is no match between the submitted keywords and the searched profiles (paragraphs 59-62), at the very least lack of returned result would be also an indication of an "empty intersection". The Appellant is not specific of what type of indication is used, therefore again term "indication" is allotted the broadest, reasonable interpretation, wherein an "indication" is just a sign which allows a user to realize what is the result of the most recent action, for instance comparison or search.

Examiner's Answer of November 16, 2007, pp. 9-10.

The Examiner's response relies on the incorrect assumption that a mathematical intersection of sets can not be empty. As shown above, the mathematical intersection of sets can be empty. Therefore, the Examiner's assertions in this regard are incorrect.

Additionally, as provided in the appeal brief, *Siefert* does not teach determining whether an intersection of sets is empty. The disclosure simply does not exist in *Siefert*. Therefore, regardless of how broadly the Examiner interprets the word "indication," *Siefert* does not teach providing an indication, as claimed, because the claimed indication is responsive to the determination of the intersection of the claimed sets. Additionally, *Siefert* does not teach that the data store does not include data items in the selected set, in the manner recited in claim 1.

Therefore, *Siefert* does not teach, "responsive to a determination that the intersection of the selected set and the profile set is empty, providing an indication that the data store does not include data items in the selected set." Accordingly, *Siefert* does not anticipate claim 1.

IV. Refutation of the Examiner's Fourth Response

In response to Appellant's argument that "*Asherman* does not teach or suggest the claimed featured of, "creating a profile of data store, the profile comprising a profile rule defining a profile set, wherein the profile set comprises a third set of one or more data items in accordance with the profile rule", the Examiner states that:

> The Examiner agrees that Asherman does not teach the above-disclosed limitations, however it is important to note that this is obviousness type of rejection, and Asherman's teaching is used as a secondary prior art. Consequently, the above-disclosed limitations are taught by Siefert (primary prior art), and therefore Asherman does not need to teach those features.

Examiner's Answer of November 16, 2007, p. 10.

The Examiner appears to misunderstand the thrust of Appellants' argument with respect to the second ground of rejection. Section B.1. of the appeal brief submits that *neither Siefert* nor *Asherman* teach or suggest the claim feature at issue; and, for that reason, the proposed combination of references, considered as a whole, does not teach or suggest the claimed feature.

Appellants recognize that the Examiner relies on *Siefert* to teach the "creating" feature of the claims. This assertion is refuted with respect to the response to the anticipation rejection. However, to establish that the combination of *Siefert* and *Asherman*, as a whole, do not teach this

claimed feature, Appellants also submitted that *Asherman* does not teach this claimed feature.

In any case, the proposed combination of references, considered as a whole, does not teach or suggest all of the features of the claims as argued in the appeal brief. Accordingly, under the standards of *In re Lowry*, the Examiner failed to state a *prima facie* obviousness rejection against the claims.

V. Refutation of the Examiner's Fifth Response

In response to Appellant's argument that the Examiner failed to provide a proper reason to combine the references to achieve the legal conclusion of obviousness under the standards of *KSR Int'l.*, the Examiner states that:

> The Appellant's second argument has not been found persuasive. First of all, the KSR Int'l ruling does not require an explicit motivation as TSM (test, suggestion and motivation). Second of all, the Appellant in his argument omitted the most important part of motivation, mainly the need for storing those files in well organized data set". The Examiner maintains that having organized structure (not addressed by the Appellant) is a useful feature. Consequently, even assuming that KSR Int'l standards would require an explicit motivation, the motivation is still present in the discussed claim.

Examiner's Answer of November 16, 2007, p. 11.

Again, the Examiner appears to have misunderstood the thrust of Appellants' arguments. Appellants do not apply the TSM test to argue the rejection, though the rejection would fail that test. Instead, Appellants directly address the requirements of *KSR Int'l.*, particularly that "*Rejections on obviousness grounds cannot be sustained by mere conclusory statements; instead, there must be some articulated reasoning with some rational underpinning to support the legal conclusion of obviousness.*" *KSR Int'l. Co. v. Teleflex, Inc.*, No. 04-1350 (U.S. Apr. 30, 2007).

In the case at hand, the Examiner's asserted reason to combine the references is that (1) the file formats are well known and (2) a need often exists for storing those files in a database. Regarding the first statement, the Examiner has provided *no rational underpinning that this fact would compel the legal conclusion of obviousness*. Simply asserting that a fact is well known provides no reasoning whatsoever that the fact should be combined with some other fact to achieve a particular claimed invention or to achieve the legal conclusion of obviousness.

Additionally, having a "useful feature" is not a rational underpinning to achieve the legal conclusion of obviousness. In stating that a "useful feature" exists only asserts that a purported

advantage exists to combining the references.

However, under *KSR Int'l.*, stating an advantage is not enough to establish the legal obviousness of a claim. Instead, the Examiner must provide some rational underpinning to the *legal conclusion* of obviousness. In this case, the Examiner has stated the purported advantage and then assumed the legal conclusion of obviousness. A rational underpinning for the legal conclusion of obviousness is not the same as an advantage; for example, one of ordinary skill would have to recognize the purported advantage, have a reason to implement the purported advantage, and also have no reason to avoid implementing the purported advantage in order to make the connection that one of ordinary skill would make the connection between the references in the first place. Additional logic would be required to state a compelling case for the *legal conclusion* of obviousness of the claim at issue; simply reciting an "advantage" is not enough. Therefore, under the standards of *KSR Int'l.*, the Examiner failed to provide a rational underpinning to achieve the legal conclusion of obviousness. Hence, the Examiner failed to state a *prima facie* obviousness rejection against the claims.

VI. Refutation of the Examiner's Sixth Response

In response to Appellant's argument that *Asherman* is non-analogous art, the Examiner states that:

> The third argument has not been found persuasive. Even though Asherman teach database communication, this teaching is still considered an analogous art with respect to the limitation it relates to. In particular, Siefert does not teach first data set comprising numeric, string .. ; data, and Asherman cures this deficiency by teaching missing limitation. It is important to note however, that this limitation does not concern retrieving data from a data store at all; instead it describes what kind of files can be stored in the data store. Consequently, those two prior arts are considered analogous since they both deal with accessing/searching files). Furthermore, it is important to note that the formats of the files listed by the Appellant are well in the art.

Examiner's Answer of November 16, 2007, pp. 11-12.

The critical determination in the *Oetiker* test for non-analogous art is not whether *Asherman* is analogous to *Siefert*, but rather whether *Asherman* is analogous *to claim 2*. As shown below, *Asherman* is not analogous to claim 2.

In light of the requirements of *In re Oetiker*, the Examiner's characterizations of *Asherman* and claim 2 are overly broad to establish that *Asherman* is in the same field of endeavor as claim 2. For example, the court in *In re Oetiker* stated that:

> The Examiner stated that "since garments commonly use hooks for securement", a person faced with the problem of unreliable maintenance of the pre-assembly configuration of an assembly line metal hose clamp would look to the garment industry art.

In re Oetiker, 977 F.2d 1443 at 1446.

The Examiner in *In re Oetiker* attempted to use substantially the same argument as the present Examiner. The argument is as follows: Because the reference and the claim both deal with the same broad class of problem, the reference is in the same field of endeavor as the claimed invention. However, the Court of Appeal for the Federal Circuit specifically provides that this argument is incorrect:

> *It has not been shown that a person of ordinary skill, seeking to solve a problem of fastening a hose clamp, would reasonably be expected or motivated to look to fasteners for garments.* The combination of elements from nonanalogous sources, in a manner that reconstructs the Appellant's invention only with the benefit of *Hind*sight, is insufficient to present a *prima facie* case of obviousness. There must be some reason, suggestion, or motivation found in the prior art whereby a person of ordinary skill in the field of the invention would make the combination. That knowledge can not come from the Appellant's invention itself.

Id. at 1447 (emphasis supplied).

The court ruled that the Examiner failed to show that a person of ordinary skill solving a problem of fastening *hose clamps* would reasonably be expected or motivated to look to a reference dealing with *fasteners for garments*. *Even though both technologies are in the same broad field of fastening objects, the reference was still considered to be non-analogous art*.

In the case at hand, *Asherman* is directed to the field of remote database communication. In contrast, claim 2 is directed to the field of in the field of searching data stores. Appellants submit that these fields are even more distinct from each other than the field of hose clamps and the field of fasteners for garments. In fact, the fields of hose clamps and fasteners for garments are *more* closely related because both are fasteners. In stark contrast, the invention of claim 2 is more divergent from the field of *Asherman*. Therefore, under the standards of *In re Oetiker*, *Asherman* is non-analogous art to claim 2, notwithstanding the Examiner's assertions to the contrary.

Accordingly, under the standards of *KSR Int'l.*, no rational underpinning exists to achieve the legal conclusion that claim 2 is obvious in view of the references. Hence, the Examiner failed to state a *prima facie* obviousness rejection against claim 2.

VII. Refutation of the Examiner's Seventh Response

In response to Appellant's argument that "Kolovson does not teach or suggest the claimed featured of, "creating a profile of data store, the profile comprising a profile rule defining a profile set, wherein the profile set comprises a third set of one or more data items in accordance with the profile rule," the Examiner states that:

> The Examiner agrees that Kolovson does not teach the above-disclosed limitations, however it is important to note that this is obviousness type of rejection, and Kolovson's teaching is used as a secondary prior art. Consequently, the above-disclosed limitations are taught by Siefert (primary prior art), and therefore Kolovson does not need to teach those features.

Examiner's Answer of November 16, 2007, p. 12.

The Examiner appears to misunderstand the thrust of Appellants' argument with respect to the second ground of rejection. Section D.1. of the appeal brief establishes that *neither Siefert* nor *Kolovson* teach or suggest the claim feature at issue; and, for that reason, the proposed combination of references, considered as a whole, does not teach or suggest the claimed feature.

Appellants recognize that the Examiner relies on *Siefert* to teach the "creating" feature of the claims. This assertion is refuted with respect to the response to the anticipation rejection. However, to establish that the combination of *Siefert* and *Kolovson*, as a whole, do not teach this claimed feature, Appellants also established that *Kolovson* does not teach this claimed feature.

In any case, the proposed combination of references, considered as a whole, does not teach or suggest all of the features of the claims. Accordingly, under the standards of *In re Lowry*, the Examiner failed to state a *prima facie* obviousness rejection against the claims.

VIII. Refutation of the Examiner's Eighth Response

In response to Appellant's argument that *Kolovson* is non-analogous art, the Examiner states that:

> The Examiner disagrees with the second argument. As stated by the Appellant, Kolovson's art is directed to a database crash recovery, and the

> Appellant's invention concerns extracting information from selected sets of data within a data store. The Examiner maintains however that Kolovson's teaching is an analogous art because at the very least a database crash recovery is associated with extracting information from selected sets, in order to back it up or to access pieces of data that might be lost.

Examiner's Answer of November 16, 2007, p. 13.

The Examiner asserts that *Kolovson* is analogous to claim 14 because, "at the very least a database crash recovery is associated with extracting information from selected sets." However, the Examiner has not applied the test of *In re Oetiker* (Appellants also challenge this assertion as unsupported and incorrect.) The *Oetiker* determination is not whether the Examiner can twist a reference to fit some aspect of a claim; rather, the *Oetiker* determination asks a basic question: Is the field of the reference the same as the field of the claim. Such a determination is plain and straightforward, not based on an interpretation of *what some aspect of claim 14 could occur* during a database crash, which is what the Examiner has effectively stated.

In light of the requirements of *In re Oetiker*, the Examiner's characterizations of *Kolovson* and claim 2 are overly broad to establish that *Kolovson* is in the same field of endeavor as claim 2. For example, the court in *In re Oetiker* stated that:

> The Examiner stated that "since garments commonly use hooks for securement", a person faced with the problem of unreliable maintenance of the pre-assembly configuration of an assembly line metal hose clamp would look to the garment industry art.

In re Oetiker, 977 F.2d 1443 at 1446.

The Examiner in *In re Oetiker* attempted to use substantially the same argument as the present Examiner. The argument is as follows: Because the reference and the claim both deal with the same broad class of problem, the reference is in the same field of endeavor as the claimed invention. However, the Court of Appeal for the Federal Circuit specifically provides that this argument is incorrect:

> *It has not been shown that a person of ordinary skill, seeking to solve a problem of fastening a hose clamp, would reasonably be expected or motivated to look to fasteners for garments.* The combination of elements from nonanalogous sources, in a manner that reconstructs the Appellant's invention only with the benefit of *Hind*sight, is insufficient to present a *prima facie* case of obviousness. There must be some reason, suggestion, or motivation found in the prior art whereby a person of ordinary skill in the field of the invention would make the combination. That knowledge can not

come from the Appellant's invention itself.

Id. at 1447 (emphasis supplied).

The court ruled that the Examiner failed to show that a person of ordinary skill solving a problem of fastening *hose clamps* would reasonably be expected or motivated to look to a reference dealing with *fasteners for garments*. *Even though both technologies are in the same broad field of fastening objects, the reference was still considered to be non-analogous art*.

In the case at hand, *Kolovson* is directed to the field of remote database crash recovery. In contrast, claim 2 is directed to the field of in the field of searching data stores. Appellants submit that these fields are even more distinct from each other than the field of hose clamps and the field of fasteners for garments. In fact, the fields of hose clamps and fasteners for garments are *more* closely related because both are fasteners. In stark contrast, the invention of claim 14 is more divergent from the field of *Kolovson*. Therefore, under the standards of *In re Oetiker*, *Kolovson* is non-analogous art to claim 14, notwithstanding the Examiner's assertions to the contrary. Accordingly, under the standards of *KSR Int'l.*, no rational underpinning exists to achieve the legal conclusion that claim 2 is obvious in view of the references. Hence, the Examiner failed to state a *prima facie* obviousness rejection against claim 14.

Note that the Examiner's assertion that, "at the very least a database crash recovery is associated with extracting information from selected sets" is irrelevant to the determination of non-analogous art. The Examiner's assertion is irrelevant because the question is whether the field of the claimed invention is in the field of the reference, not whether an aspect of the claim can be used in a particular field. This result is confirmed by the court in *In re Oetiker*, in that both the claim and the reference were directed to fasteners (one could be used with another), the reference was nevertheless non-analogous to the claim.

Because *Kolovson* is non-analogous art, the Examiner is not entitled to use *Kolovson* as a reference when asserting an obviousness rejection. Accordingly, the Examiner failed to state a *prima facie* obviousness rejection against the claims.

IX. Refutation of the Examiner's Ninth Response

In response to Appellant's argument that "Jiang does not teach or suggest the claimed featured of, "creating a profile of data store, the profile comprising a profile rule defining a profile

set, wherein the profile set comprises a third set of one or more data items in accordance with the profile rule," the Examiner states that:

> The Examiner agrees that Jiang does not teach the above-disclosed limitations, however it is important to note that this is obviousness type of rejection, and Jiang's teaching is used as a secondary prior art. Consequently, the above-disclosed limitations are taught by Siefert (primary prior art), and therefore Jiang does not need to teach those features:

Examiner's Answer of November 16, 2007, pp. 13-14.

The Examiner appears to misunderstand the thrust of Appellants' argument with respect to the second ground of rejection. Section E. of the appeal brief submits that *neither Siefert* nor *Jiang* teach or suggest the claim feature at issue; and, for the stated reason, the proposed combination of references, considered as a whole, does not teach or suggest the claimed feature.

Appellants recognize that the Examiner relies on *Siefert* to teach the "creating" feature of the claims. However, Appellants refuted the assertion that *Siefert* teaches the "creating" feature with respect to the response to the anticipation rejection. However, to establish that the combination of *Siefert* and *Jiang*, as a whole, do not teach this claimed feature, Appellants also submit that *Jiang* does not teach this claimed feature, as also identified in the appeal brief.

Therefore, the proposed combination of references, considered as a whole, does not teach or suggest all of the features of the claims. Accordingly, under the standards of *In re Lowry*, the Examiner failed to state a *prima facie* obviousness rejection against the claims.

CONCLUSION

As shown above, the Examiner has failed to state valid rejections against any of the claims. Therefore, Appellants request that the Board of Patent Appeals and Interferences reverse the rejections. Additionally, Appellants request that the Board direct the Examiner to allow the claims.

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EVIDENCE APPENDIX

The attached article, Bennet, Set and Probability Theory, Lecture Note 1, Massachusetts Institute of Technology department of Economics, Spring 2006, is submitted only to show that the Examiner's assertion regarding set theory is wrong. This article can be found at: ocw.mit.edu/NR/rdonlyres/Economics/14-30Spring-2006/9D2A63C6-DD02-49B2-A4E3-736F5ACC314B/0/l1.pdf.

LECTURE NOTE 1 *

SET AND PROBABILITY THEORY

MIT 14.30 SPRING 2006

HERMAN BENNETT

1 Set Theory

1.1 Definitions and Theorems

1. Experiment: any action or process whose outcome is subject to uncertainty.

2. Sample Space: collection of all possible outcomes (or elements) of the experiment (set S). [Finite vs. Infinite; Discrete vs. Continuous]

*Caution: These notes are not necessarily self-explanatory notes. They are to be used as a complement to (and not as a substitute for) the lectures.

3. Event: collection of elements (subset A, B, etc.) contained in the sample space (S).

4. $s \in S$: The outcome s belongs to the sample space S. The contrary is defined by the symbol $\notin$.

5. $\emptyset = \{\}$: Denotes the empty set (the set of no elements). It also defines the set of elements of an impossible event; *e.g.*: the event 'generating a negative number' when rolling a die.

6. Union: The union of event A and event B, denoted $A \cup B$, is the collection (or set) of elements that belong to either A or B or both. $[A \cup B = \{x : x \in A \text{ or } x \in B\}]$

   Properties: $A \cup A = A$; $A \cup S = S$; $A \cup \emptyset = A$.

7. Intersection: The intersection of event A and event B, denoted $A \cap B$, is the collection (or set) of elements that belong to A and B. $[A \cap B = \{x : x \in A \text{ and } x \in B\}]$

   Properties: $A \cap A = A$; $A \cap S = A$; $A \cap \emptyset = \emptyset$.

8. Complement: The complement of event A, denoted A^c (or equivalently A'), is the set of all elements that are not in A. $[A^c = \{x : x \notin A\}]$

   Properties: $(A^c)^c = A$; $\emptyset^c = S$; $S^c = \emptyset$; $A^c \cup A = S$;
   $A^c \cap A = \emptyset$.

9. $A \subset B$: The event A is contained in event B, $A \subset B$, if every element of A also belongs to B.

   Properties: – If $A \subset B$ and $B \subset A \Rightarrow A = B$;
   – If $A \subset B$ and $B \subset C \Rightarrow A \subset C$;
   – $\emptyset \subset A$, for any event A.

10. Disjoint: Event A and event B are disjoint, or mutually exclusive, if A and B have no outcome in common. [$A \cap B = \emptyset \Leftrightarrow A$ and B are disjoint events]

11. Exhaustive: Event A and event B are exhaustive if their union is S. [$A \cup B = S \Leftrightarrow$ A and B are exhaustive events]

12. Finally, some additional results (HOMEWORK: think of them in terms of Venn diagrams):
    - Commutativity: $A \cup B = B \cup A$; $A \cap B = B \cap A$.
    - Associativity: $A \cup (B \cup C) = (A \cup B) \cup C$; $A \cap (B \cap C) = (A \cap B) \cap C$.
    - Distributive Laws: $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$; $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$.
    - DeMorgan's Laws: $(A \cup B)^c = A^c \cap B^c$; $(A \cap B)^c = A^c \cup B^c$.

2 Probability Theory

2.1 Definition of Probability

How likely is it for event A to occur? This concept is represented by the *probability that event A will take place*, which is denoted by $P(A)$ and can take any value from 0 to 1.

The mathematical definition of probability function is based on 3 axioms, which are based on our intuitive notion of probability. $[P() : \{\text{set of all possible events}\} \rightarrow [0,1]]$

– Axiom 1: For any event A, P(A)$\geq$ 0 (nonnegative).
– Axiom 2: P(S)=1.
– Axiom 3: For any sequence of disjoint sets $A_1, A_2, ..., A_n$, $P(A_1 \cup A_2 \cup ... A_n) = \sum_{i=1}^{n} P(A_i)$ where n is the total number of disjoint sets in the sequence.

Properties (for events A and B):
– $P(A) = 1 - P(A^c)$; $P(A \cup B) = P(A) + P(B) - P(A \cap B)$; $P(\emptyset) = 0$;
– If A and B are disjoint $\Rightarrow P(A \cap B) = 0$;
– If $A \subset B \Rightarrow P(A) \leq P(B)$.

Example 2.1. Show that $P(A \cup B) = P(A) + P(B) - P(A \cap B)$.

2.2 Counting Techniques to Compute $P(A)$ when each Possible Outcome is Equally Likely – (same Probability)

$$P(A) = \frac{N(A)}{N} \tag{1}$$

Where N is the number of outcomes contained in S and $N(A)$ is the number of outcomes contained in event A.

When the sample space is small and the outcomes are equally likely (same probability), just count. For example, rolling a die: $N = 6$, $P(3) = \frac{1}{6}$. If the case you are dealing with is not so simple, you can use the following techniques to count.

1. General Product Rule: If a process has multiple stages (call the number of stages k) and if stage i can be completed in n_i ways, regardless of which outcomes occur in earlier stages, then the process itself can be completed in $\mathbf{n_1 n_2 ... n_k}$ ways. Note that the choices are not necessarily the same in each stage (although they could be).

   Example 2.2. Assume a box that contains 7 balls of different colors. How many ways are there to take 3 balls from the box, if each ball taken is immediately returned to the box?

2. Permutations: Suppose the outcome is constructed by selecting k objects from a total of n and *without replacement*. The total number of permutations (which means that *order matters*) is $\mathbf{n(n-1)...(n-k+1)}$. General formula: $\mathbf{P_{k,n} = \frac{n!}{(n-k)!}}$ [Following Example 2.2: $P_{3,7} = \frac{7!}{(7-3)!} = 7 \cdot 6 \cdot 5$]

Example 2.3. How many ways are there to rank 4 different dogs? How many ways are there to rank 4 different dogs out of a total of 10 dogs?

3. Combinations: Now assume the outcome is constructed in the same way as before: selecting k objects from a total of n and *without replacement.* The total number of combinations (which means that *order does not matter*) is: $\mathbf{C_{k,n}} = \binom{\mathbf{n}}{\mathbf{k}} = \frac{\mathbf{n!}}{\mathbf{k!(n-k)!}}$. The symbol $\binom{n}{k}$ is read "n choose k"; is the number of ways a group of k objects can be selected from a collection of n objects. [Following Example 2.2: $C_{3,7} = \frac{7!}{3!\cdot(7-3)!}$]

Example 2.4. How many possible combinations of 3 books are there in a set of 5 books? How many possible combinations of 5 books are there in a set of 5 books? (Note the difference from permutation.)

Wrap-up: When simple counting is not practical, we use techniques 1-3 to compute N, the number of outcomes contained in the sample space, and to compute N(A), the number of outcomes contained in event A. With this information we can compute $P(A)$.

Example 2.5. A deck of 52 cards has 4 aces. Assume you give 13 cards to each of the 4 players. What is the probability that each player gets exactly 1 ace?

Example 2.6. A fair coin is tossed 7 times. What is the probability of obtaining 3 heads? What is the probability of obtaining at most 3 heads?

2.3 Conditional Probability

We use probabilities because we are uncertain about the exact outcome of an experiment. However, this does not mean that we are completely ignorant about the process. The belief about the likelihood of an event, $P(A)$, is based on the information at hand when the assignment of probability is made. New information can be available, which could make us modify our belief (probability). Conditional Probability, $P(A|B)$, is the name given to the new belief after receiving the new information, in this case that event B occurred. [1]

$$\text{Definition: } P(A|B) = \frac{P(A \cap B)}{P(B)}, \quad \text{for } P(B) > 0 \tag{2}$$

Note that:

- $P(A|B)P(B) = P(B|A)P(A)$.
- If events $A_1, A_2, ...A_k$ are disjoint and exhaustive, then:

$$P(A_1|B) + P(A_2|B)...+ = 1 \text{ and}$$

$$\sum_{i=1}^{k} P(B|A_i)P(A_i) = P(B) \text{ (Law of Total Probability).}$$

Bayes Theorem. Let the events $A_1, A_2, ...A_k$ be disjoint and exhaustive events in the sample space S, such that $P(A_i) > 0$, and let B be an event such that $P(B) > 0$. Then,

$$P(A_i|B) = \frac{P(B|A_i)P(A_i)}{\sum_{i=1}^{k} P(B|A_i)P(A_i)} \quad \left(= \frac{P(A_i \cap B)}{P(B)} \quad \frac{\text{cond. prob.}}{\text{law of total prob.}} \right) \tag{3}$$

This way of updating the probability of event A is usually called *Bayesian updating.*

[1] $P(A|B)$ and $P(A)$ are also called *posterior* and *prior*, respectively.

Example 2.7. There is a new music device in the market that plays a new digital format called MP∞. Since it's new, it's not 100% reliable. You know that 20% of the new devices don't work at all, 30% last only for 1 year, and the rest last for 5 years. If you buy one and it works fine, what is the probability that it will last for 5 years?

2.4 Independence

Two events A and B are said to be independent if $P(A|B) = P(A)$; otherwise they are dependent.

- For example, tossing a fair coin twice. The probability of getting H or T on the second toss does not depend on whether you got H or T in the first. Another way to see this: the result of the first toss does not provide any additional information about the result of the second one: $P(A|B) = P(A)$.

- If A and B are independent, then $P(A \cap B) = P(A)P(B)$ (by definition of conditional probability).

- If A and B are independent, then A and B^c are also independent $[P(A \cap B^c) = P(A)P(B^c)]$.

- General definition of independence between 2 or more events: Events $A_1, A_2, ..., A_n$ are mutually independent if, for all possible subcollections of $k \leq n$ events: $P(A_i \cap A_j \cap ... A_k) = P(A_i)P(A_j)...P(A_k)$.

Example 2.8. Events associated with the experiment of rolling a die: $A = \{2, 4, 6\}$ $B = \{1, 2, 3, 4\}$ $C = \{1, 2, 4\}$. Are events A and B independent? What about A and C?